Values of Fluents

An AOG is a model of all valid plans to achieve a fluent-change. This formulation of values establishes a total order on the parse-graphs.

Learning Values

Denote internal fluents as $F(s) \triangleq x$, external fluents as $F(\hat{s}) \triangleq u$, and motor control as a. A task plan is the optimal motion sequence that maximizes value.

$$a^*_{[1:t]} = \arg\max_{a_{[1:t]}} V(a_{[1:t]})$$

The search for a value function is facilitated by the expansion below.

$$\frac{\partial V}{\partial a_{[1:t]}} = \underbrace{\frac{\partial V}{\partial x}}_{\text{"why"}} \underbrace{\frac{\partial x}{\partial u}}_{\text{"how"}} \underbrace{\frac{\partial u}{\partial a_{[1:t]}}}_{\text{"what"}}$$

To solve the first part $\frac{\partial V}{\partial x}$, we define a Lagrangian,

$$L(\dot{x}, x, t) = E_{kinetic} - E_{potential} = Cost(\dot{x}) + V(x)$$

minimizing the functional,

$$x^*(t) = \arg\min_{x(t)} \int_a^b L(\dot{x}, x, t)\ dt$$

to use the Euler-Lagrange equation.

$$\frac{d}{dt}\Big(\frac{\partial L}{\partial \dot{x}}\Big) = \frac{\partial L}{\partial x}$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{d}{d\dot{x}} Cost(\dot{x}) \qquad \frac{\partial L}{\partial x} = \frac{d}{dx} V(x)$$

$$\frac{d}{dt}\Big(\frac{\partial}{\partial \dot{x}} Cost(\dot{x})\Big) = \frac{d}{dx} V(x)$$

Where the cost of a fluent-change is defined as the expected cost of actions that can achieve it.

$$Cost(\dot{x}) \triangleq E_{u|\dot{x}}[Cost(u)] = \int_{\Omega_u} Cost(u)\ P(u|\dot{x})\ du$$

And in turn, the cost of an action is recursively defined as the expected cost of a fluent-change.

$$Cost(u) \triangleq E_{\dot{x}|u}[Cost(\dot{x})] = \int_{\Omega_{\dot{x}}} Cost(\dot{x})\ P(\dot{x}|u)\ d\dot{x}$$

We hypothesis the interlinked cost functions converge to a fixed point equilibrium, like PageRank.

Either way,

$$\frac{\partial V}{\partial x} = \frac{d}{dt}\Big(\frac{\partial}{\partial \dot{x}} \int_{\Omega} Cost(u)\ P(u|\dot{x})\ du\Big)$$

AOG Task Plan

A fluent-vector describes all the fluents at some time, $F(s) \in \mathbb{R}^N$. The state-space for planning is a set of fluent-vectors, $\{F(s)\}_s$. A set of actions A can transition between states.

The And-Or Graph G is a policy, and any parse-graph pg is a valid plan. There are many plans pg to achieve the same goal: $\Omega = \{pg \mid pg \in G\}$.

The optimal action sequence is driven by an optimal plan, which is defined as the parse-graph with minimum cost.

$$a^*_{[1:t]} = pg^* = \underset{pg \in \Omega}{\arg\min}\ Cost(pg) = \underset{pg \in \Omega}{\arg\max}\ V(pg) = \underset{a_{[1:t]} = pg \in \Omega}{\arg\max}\ V(a_{[1:t]})$$

Every fluent-vector $F(s)$ has a value $V_{F(s)} \in \mathbb{R}$.

Every change in fluent $\partial F = (\partial f_1, ..., \partial f_N)$ has a cost $Cost(\partial F) \in \mathbb{R}^+$.